12/3 Recitation

1. Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$.
   (a) Show that $\mathcal{B}$ is a basis of $\mathbb{R}^3$.

   (b) Find the $\mathcal{B}-$coordinate vector of $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

2. Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + 4y \\ x + y \end{bmatrix}$ (expressed in the standard basis).
   (a) Write down the standard matrix A for T (written in the standard basis)

   (b) Write down the associated matrix D of T written in the basis $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$.

   (c) Diagonalize A without finding eigenvalues or eigenvectors.

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation with associated matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$ (written in the standard basis). Find a basis for which the associated matrix is diagonal.

4. In class we saw that if $\{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ is a basis for $\mathbb{R}^n$, then every vector in $\mathbb{R}^n$ can be written *uniquely* as a linear combination of the basis vectors; the key was linear independence. Show that if $\{\mathbf{b}_1, \ldots, \mathbf{b}_k\}$ spans $\mathbb{R}^n$, but is not linearly independent, then every vector in $\mathbb{R}^n$ can be written as a linear combination of $\mathbf{b}_1, \ldots, \mathbf{b}_k$ in infinitely many ways.